

The audio for the last 6 mins of the video lecture did not get recorded. Here is what I explained in those last minutes:

- We use dynamic programming to compute $P_Y(x,y)$ for each label subset Y . In the previous slide, we saw that the two inner summations of the RHS of $P(x,y)$ was equal to the polynomial $P_Y(x,y)$.
- Now, the outer summation of the RHS of $P(x,y)$ is over all subsets X and Y is $[k]\setminus X$. Therefore, the outer summation is essentially over all subsets Y .
- Thus, once we evaluate all $P_Y(x,y)$ for all label subsets Y , we can evaluate $P(x,y)$.
- Time taken is polynomial for evaluating each $P_Y(x,y)$ and there are 2^k possible Y 's. So total time to evaluate $P(x,y)$ is $2^k \text{ poly}(n)$.
- Last slide: Thus, we obtain a randomized FPT algorithm for k -path, running in $2^k \text{ poly}(n)$ time. The randomization is due to application of the Schwartz-Zippel Lemma and the bulk of the running time is because of evaluating the Labeled Walk polynomial (using weighted Inclusion-Exclusion).
- In this part, we have looked at 2 important algebraic tool, Inclusion-Exclusion and identically zero polynomial testing, in order to design FPT algorithms.
- Algebraic tools like these are often used in Parameterized Complexity to design algorithms. In particular, in recent times many state of the art algorithms are based on algebraic techniques.