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**INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR**  
**Randomized Algorithm Design: Second Class Test 2018-19**

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**Date of Examination: 3 April 2019**

**Session (FN/AN): Class Test II**

**Duration: 1 hours**

**Full Marks: 20**

**Subject No: CS60029**

**Subject: Randomized Algorithm Design**

**Department/Center/School: COMPUTER SCIENCE AND ENGINEERING**

**Specific charts, graph paper, log book etc., required: NO**

**Special instruction (if any): NA**

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**Answer all question.**

1. Prove, using probabilistic method, that any graph has a bipartite sub-graph containing at least half the total number of edges.

**[5 Marks]**

2. Prove that, for any prime number  $p$ , the family  $\mathcal{H} = \{h_{a,b} : \mathbb{Z}_p \rightarrow \mathbb{Z}_p, h_{a,b}(x) = ax + b \pmod{p} : a, b \in \mathbb{Z}_p\}$  of hash functions is NOT 3 universal.

**[5 Marks]**

3. Using Lovasz Local Lemma, prove that, if

$$4 \binom{\ell}{2} \binom{n}{\ell-2} 2^{1-\binom{\ell}{2}} \leq 1$$

then it is possible color the complete graph  $K_n$  on  $n$  vertices with two colors so that there is no monochromatic  $K_\ell$  subgraph of  $K_n$ .

**[10 Marks]**