
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Randomized Algorithm Design: First Class Test 2018-19

Date of Examination: 23 January 2019

Session (FN/AN): Class Test I

Duration: 1 hours

Full Marks: 20

Subject No: CS60029

Subject: Randomized Algorithm Design

Department/Center/School: COMPUTER SCIENCE AND ENGINEERING

Specific charts, graph paper, log book etc., required: NO

Special instruction (if any): NA

Answer all question.

1. Let \mathcal{X} be a discrete random variable with finite expectation. Suppose support of \mathcal{X} be \mathbb{N} . Then prove that $\mathbb{E}[\mathcal{X}] = \sum_{i=1}^{\infty} \Pr[\mathcal{X} \geq i]$.

[5 Marks]

2. Let \mathcal{X} and \mathcal{Y} be two random variables defined on the same underlying probability space $(\Omega, \mathcal{F} = 2^{\Omega}, \mathcal{P})$ where Ω is a finite set. Then prove that $\mathbb{E}[\mathbb{E}[\mathcal{X}|\mathcal{Y}]] = \mathbb{E}[\mathcal{X}]$.

[5 Marks]

3. Fix any input sequence of n integers to the quick sort algorithm. Let \mathcal{X} be the random variable denoting the number of comparisons the quick sort algorithm makes on the input sequence. Then prove that $\text{var}(\mathcal{X}) = \mathcal{O}(n^2)$.

[10 Marks]