INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Algorithmic Game Theory: Mid-Semester Examination 2018-19

Date of Examination: 16 September 2019
Duration: 2 Hours
Full Marks: 40
Subject No: CS60025
Subject: Algorithmic Game Theory
Department/Center/School: COMPUTER SCIENCE AND ENGINEERING

Special instruction (if any):

(i) You do not need to prove anything that is already proven in the class.

(ii) If you want to use any result which is not proved in class, you should prove it.

Answer question number 6 and any 3 questions out of questions 1 to 5 (4 questions in total).

1. Let α be a correlated equilibrium of a matrix game $\mathcal{A}$ with m rows and n columns. Prove that $u_1(\alpha)$ (the utility of the row player) is equal to the value of the game in mixed strategies.

[10 Marks]

2. Assume the first price auction scenario with 2 sellers with respective valuations v_1 and v_2 for the item. That is the seller with lower bid wins the auction, the winner sells the item, and receives his/her bid as his/her payment. If both players bid the same amount, then winner is chosen uniformly randomly. Does there exist any PSNE of the induced game? If yes, then compute all PSNEs of the induced game.

[10 Marks]

3. Consider the *online selection* problem: we get to see n items $a_1, \ldots, a_n$ with respective values $v_1, \ldots, v_n \in \mathbb{R}$ according to the order $1, \ldots, n$. At the i-th step, we get to know the value v_i of the i-th item a_i and we need to decide, in the i-th step itself, whether to select a_i or reject a_i. An item once selected (or rejected respectively) can never be rejected (or selected respectively). Let $\mathcal{A}$ be any randomized algorithm to select exactly one item in the above framework. Then, prove that there is an instance $\mathcal{I}$ of the problem such that

$$\mathbb{E}[\mathcal{A}(\mathcal{I})] \leqslant \frac{1}{n} \max_{i \in [n]} v_i$$

where $\mathcal{A}(\mathcal{I})$ is the value of the item selected by the algorithm.

[10 Marks]

4. Prove that the selfish load balancing game with n jobs with corresponding weights $w_1, \ldots, w_n$, and m identical machines is a potential game with the following potential function.

$$\Phi = \frac{1}{2} \sum_{j=1}^{m} \ell_j^2$$

where ℓ_j is the load of machine j.

[10 Marks]

5. Consider a class of bimatrix games where the utility of every player is in $\left[0, \frac{1}{2}\right]$. Design a polynomial time algorithm to compute a $\frac{1}{4}$ approximate equilibrium for the above class of games.

[10 Marks]

6.⋆ *Consider the following strategic form game: We have n players (assume n is an odd integer). The strategy set of every player is $\{1, 2, \ldots, m\}$ for some fixed positive integer $m > 1$. Every player $i \in [n]$ is associated with an integer $a_i \in \{1, 2, \ldots, m\}$. In a strategy profile $(s_1, \ldots, s_n) \in \{1, 2, \ldots, m\}^n$, the cost $C_i(s_1, \ldots, s_n)$ of player $i \in [n]$ is $|a_i - \text{med}(s_1, \ldots, s_n)|$ where med$(s_1, \ldots, s_n)$ is the median of $(s_1, \ldots, s_n)$; observe that, since n is an odd integer, we have a unique median. Recall that players' utilities are minus of their corresponding costs. Prove that $(a_1, a_2, \ldots, a_n)$ is a weakly dominant strategy equilibrium of the above strategic form game.*

[10 Marks]

Best of luck