**INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR**
**Algorithmic Game Theory: End-Semester Examination 2019-20**

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**Date of Examination: 18 November 2019**
**Duration: 3 Hours**
**Full Marks: 50**
**Subject No: CS60025**
**Subject: Algorithmic Game Theory**
**Department/Center/School: COMPUTER SCIENCE AND ENGINEERING**

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**Special instruction (if any):**

**(i) You do not need to prove anything that is already proven in the class.**

**(ii) If you want to use any result which is not proved in class, you should prove it.**

**(iii) For questions involving numericals, it is enough to write answer as an expression and not compute it. For example, it is perfectly fine to write $19 * 3^8 2/11$ and not compute it using calculator.**

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## Answer question number 6 and any 4 questions out of questions 1 to 5 (5 questions in total).

1. In a stable matching instance with sets $\mathcal{A}$ and $\mathcal{B}$ of $n$ men and women, suppose $\mathcal{M}_1$ and $\mathcal{M}_2$ be two stable matchings. Define $\mathcal{M}_3 = \{(a, b) \in \mathcal{A} \times \mathcal{B} : \mathcal{M}_1(a) = \mathcal{M}_2(a) = b$ or $b = \mathcal{M}_1(a) \succ_a \mathcal{M}_2(a)$ or $b = \mathcal{M}_2(a) \succ_a \mathcal{M}_1(a)\}$; that is, in $\mathcal{M}_3$, every man $a \in \mathcal{A}$ gets his better partner between $\mathcal{M}_1(a)$ and $\mathcal{M}_2(a)$. Prove that $\mathcal{M}_3$ is also a stable matching.

**[10 Marks]**

2. Consider a scenario with a set of five sellers selling identical items with respective valuations $v_1 = 23, v_2 = 15, v_3 = 11, v_4 = 8, v_5 = 2$ and one buyer. Compute VCG payments in each of the following cases.

   (i) The buyer wishes to buy 4 items and each seller can supply at most one items.

   (ii) The buyer wishes to buy 4 items and each seller can sell at most 2 items.

**[5+5 Marks]**

3. Prove or disprove the following with explanation.

   (i) A correlated equilibrium of a bimatrix game can be computed in polynomial time.

   (ii) Conclude form (i) that an MSNE of a bimatrix game can be computed in polynomial time.

**[5+5 Marks]**

4. (i) Consider a Bayesian game with 2 players, each player having a type set of cardinality 5, and each player has a strategy set of cardinality 7. Compute the number of players, and the cardinality of each player's strategy set in the corresponding Selten game. Explain your answer.

   (ii) Consider an extensive form game with 2 players. Player 1 plays her action first, then the player 2 plays her action, and then both the players receive their utilities. Suppose player 1 has 7 actions and player 2 has 11 actions. Also each information set is singleton. Compute the number of players, and the cardinality of each player's strategy set in the corresponding strategic form game. Explain your answer.

**[5+5 Marks]**

5. Let $f : \times_{i \in [n]} \Theta_i \longrightarrow \mathcal{X}$ be a social choice function. Let $\mathbb{P}$ be a belief distribution over $\times_{i \in [n]} \Theta_i$ and $u_i : \mathcal{X} \longrightarrow \mathbb{R}$ be the utility function of player $i$ which are common knowledge. Prove or disprove the following.

   (i) If $f$ is ex-post individually rational, then $f$ is interim individually rational.

   (ii) If $f$ is interim individually rational, then $f$ is ex-ante individually rational.

**[5+5 Marks]**

6.* *Prove or disprove: the top trading cycle algorithm (TTCA) for the house allocation problem is group strategy proof — that is, there does not exist any subset of players who lie in such a way that everybody in the group gets at least as good a house as TTCA allocation under truthful setting and at least one player in the group gets strictly better house.*

**[10 Marks]**

———— Best of luck ————