Assignment 3: Algorithmic Game Theory

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Submit the problems in brown color. Deadline: 30 October 2019 midnight. Submit it in my mail box in the Department.

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1. Prove revelation principle for BIC mechanisms.
2. Let $f : \Theta \longrightarrow \mathcal{X}$ be a social choice function such that we have the following for every $\theta \in \Theta$

$$\sum_{i=1}^{n} u_i(f(\theta), \theta_i) \geqslant \sum_{i=1}^{n} u_i(x, \theta_i) \ \forall x \in \mathcal{X}$$

   Show that f is ex-post efficient.

3. (Taken from an exercise in [Nar14]) Let $N = \{1, 2\}, \Theta_1 = \{a_1, b_1\}, \Theta_2 = \{a_2, b_2\}, \mathcal{X} = \{x, y, z\}$ and

$$u_1(x, a_1) = 100, u_1(y, a_1) = 50, u_1(z, a_1) = 0$$
$$u_1(x, b_1) = 50, u_1(y, b_1) = 100, u_1(z, b_1) = 40$$
$$u_2(x, a_2) = 0, u_2(y, a_2) = 50, u_2(z, a_2) = 100$$
$$u_2(x, b_2) = 50, u_2(y, b_2) = 30, u_2(z, b_2) = 100$$

   For the above environment, give an example for a social choice function for each of the following cases (EPE: Ex-Post Efficient, DSIC: Dominant Strategy Incentive Compatible, BIC: Bayesian Incentive Compatible, D: Dictatorship, ND: Non-dictatorship).

   (i) EPE, DSIC, and D
   (ii) EPE, DSIC, and ND
   (iii) Not EPE but DSIC and ND
   (iv) EPE, BIC (under suitable prior), but not DSIC
   (v) EPE but not BIC (under suitable prior)

4. Can a social choice function has more than one dictator if every player has a strict rational preference relation?
5. Consider the set of outcomes $\mathcal{X}$ to be the set of integers in the range from 0 to 100. There are n players. The type of player i is θ_i and the utility of player i is $u_i(x) = -|x - \theta_i|$ for every $x \in \mathcal{X}$. Design n social choice functions $f_1, \ldots, f_n : \times_{i \in [n]} \Theta_i \longrightarrow \mathcal{X}$ each one of which is non-dictatorship as well as DSIC.

References

[Nar14] Y. Narahari. *Game Theory and Mechanism Design*. World Scientific Publishing Company Pte. Limited, 2014.