CS 60047

Indian Institute of Technology Kharagpur

AUTUMN Semester 2019

COMPUTER SCIENCE AND ENGINEERING

CS 60047 Advanced Graph Theory

Mid-Semester Examination   Date: 20 September 2019

Full Marks: 60   Credit: 30%   Time allowed: 2 hours

INSTRUCTIONS: This exam is closed notes and closed books. This question paper has two pages. Use of calculators is allowed. ATTEMPT ALL QUESTIONS.

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1. **(10 points)**

(a) What is the smallest number of edges to be removed from K_7 so that the residual graph becomes bipartite? Justify your argument.

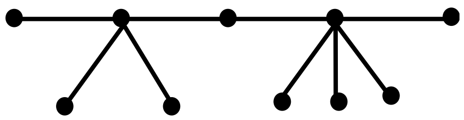
(b) Have many different labeled spanning trees are there in K_7?

(c) A tree T has one vertex of degree 6, three vertices of degree 4, and two vertices of degree 3. How many leaf nodes does T have? (4 + 3 +3)

2. **(10 points)**

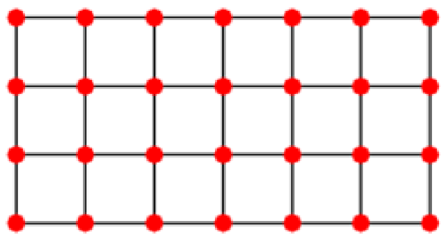
(a) Construct a labeled tree corresponding to the Prüfer code (2, 3, 2, 3, 2, 3, 2, 3). Show your steps.

(b) For the caterpillar shown below, suggest a graceful labeling of nodes. (4 + 6)



3. **(10 points)**

(a) The nodes of a grid graph, shown below, represent cities and edges denote road-segments with one-way traffic. However, some road-segments are damaged and need repair. What is the maximum number of road segments that can be repaired simultaneously so that a driver can still travel between any pair of cities? Show the traffic directions for your solution.

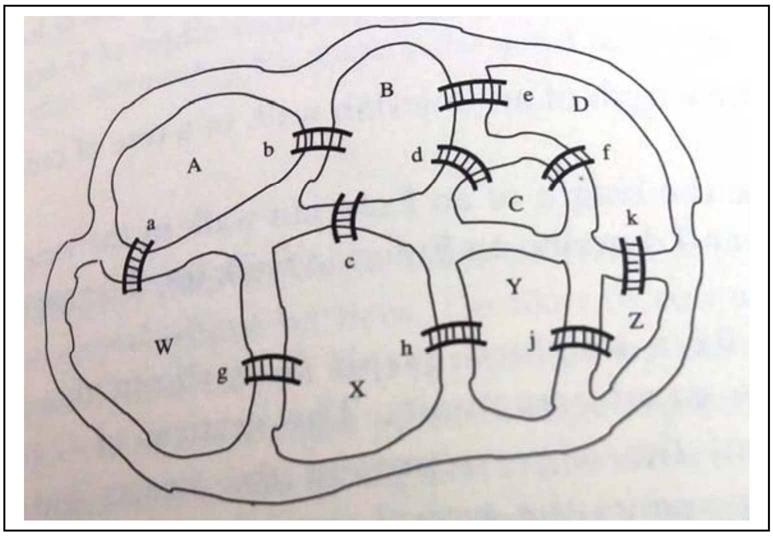


(b) Consider an undirected graph G (V, E) where each vertex denotes a tournament on five labeled vertices. Two vertices in G are connected by an edge if their corresponding tournaments differ by reversal of orientation in exactly one edge. What is the diameter of G?

(c) Show that if n is a positive odd integer, $n \geq 3$, there exists a tournament in which every vertex is a king. (3 + 3 + 4)

4. **(10 points)**

(a) In the lobby of a hotel, a waterway has been constructed that surrounds eight land areas *A, B, C, D, W, X, Y, Z* as shown below. At certain locations, ten bridges labeled *a, b, c, d, e, f, g, h, j, k,* have been built over water.



(i) Is it possible to walk over the land regions such that each bridge is crossed exactly once? If so, show the walk.

(ii) Is it possible to have a boat ride through the waterway so that the boat goes under each bridge exactly once?

(b) A road network of a locality resembles a spanning tree of Petersen graph. A postman starts from one node. He has to travel each road at least once and return to the starting node. What is the minimum cost of such a travel? Assume unity cost for each edge. Justify your answer. ((4 + 3) +3)

5. **(10 points)**

(a) Show that in a graph G, radius(G) $\leq$ diameter(G) $\leq 2 \times$ radius(G).

(b) Construct a directed graph whose degree sequence is {(4, 1), (2, 1), (1, 1), (1, 1), (1, 1), (1, 1), (0, 2), (0, 2)}, such that its underlying graph is simple and connected. (5 + 5)

6. **(10 points)**

(a) Let G be simple graph with n vertices ($n \geq 2$). What is the maximum number of edges G can have so that G has an independent vertex set of size k, $k < n$?

(b) Prove or disprove the following claim: Let v denote a node in hypercube Q_5. Consider the graph G: $Q_5 - \{v\}$. We claim that G does not admit a Hamiltonian cycle. (3 + 7)