Indian Institute of Technology Kharagpur

Computer Science and Engineering

CS 60047 Advanced Graph Theory Autumn 2019

Date: 06.9.2019 **Maximum marks = 100** Credit: 10%

Class-Test 1 **Time:** 9:15-10:45

Name:____________________________ **Roll No.**:_________________________

Instructions

A. This is an **OPEN-BOOK/OPEN-NOTES** test. Answer all questions.

B. Please write your answers on the **TEST PAPER** itself**.** Kindly use other sheets for your rough work, if needed.

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1. **(10 points)** A road network is given that resembles an undirected complete graph of six vertices (K_6); its vertices are labeled as 1, 2, 3, 4, 5, 6. The travel time between two adjacent nodes labeled i and j is $|i - j|$. A postman starts from vertex 1 and travels along each edge of the graph at least once before returning to vertex 1.

   (i) Determine a route that minimizes the total time of travel (t) of the postman;

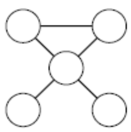
   (ii) What is the value of t?

2. **(10 points)** An undirected graph G (V, E) is given, where $|V| = 100$, and they are labeled as 1, 2, 3, ….., 99, 100. Two vertices labeled i and j are adjacent if and only if ($i \times j$) is an even number. Prove that G admits a Hamiltonian cycle.

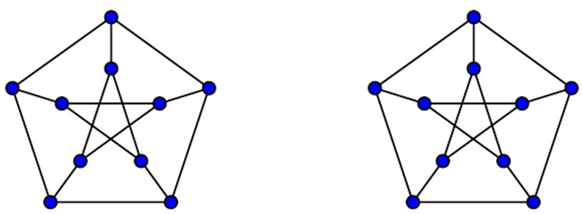
3. **(10 points)** Consider two nodes labeled a: 001011 and b: 111100 in a *de Bruijn* graph of order six. Show the sequence of intermediate vertices and edges that connects a to b via a shortest directed path.

4. **(10 points)** For every $n \geq 6$, construct a simple undirected graph that has n vertices, $(n + 2)$ edges, and exactly 6 cycles.

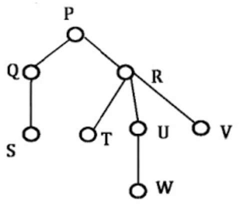
5. **(10 points)** Construct a regular graph that contains the following graph shown below, as an induced subgraph.



6. **(10 points)** Consider a graph comprising two copies of Petersen graph shown below. What is the minimum number of edges required to connect these two components so that the composite graph becomes (i) Eulerian, and (ii) Hamiltonian. Justify your answer. (5 + 5)



7. **(15 points)** (a) Consider the following tree *T* below. Can *T* be an interval graph? If yes, show the family of intervals, whose overlap graph is *T*; otherwise prove that it is impossible. (7)



(b) A pharmaceutical company has to preserve six chemicals M1, M2, ….., M6. Each chemical ought to be stored within a specified temperature range as follows: M1: (10^0F – 40^0F); M2: (20^0F – 65^0F); M3: (30^0F – 70^0F); M4: (60^0F – 100^0F); M5: (80^0F – 120^0F); M6: (90^0F – 140^0F). Determine the minimum number of fixed-temperature containers that are required to store these chemicals. Present a graph-theoretic formulation to solve this problem in polynomial time. (8)

8. **(15 points)** (a) If a tournament among *n* vertices (T_n), $n > 3$, contains a directed cycle of length *n*, then show that T_n contains a directed 3-cycle. (8)

(b) Let T_6 denote a tournament among six vertices and assume that node v is a king in T_6. We reverse the direction on each edge to obtain a reverse tournament T'_6. Construct T_6 in such a way that v becomes a king in T'_6 as well. (7)

9. **(10 points)** Consider a (7×4) rectangular grid graph G below with 28 vertices. What is the domination number of G? Justify your argument.