CS60007 Algorithm Design and Analysis 2018
Assignment 4

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Please submit the solutions of problem 1, 5, and 12. The deadline is November 12, 2018 in the class.

General Instructions

▷ Please prove correctness of every algorithm you design.

▷ If not stated otherwise, please ensure that your algorithm runs in polynomial time.

▷ If not stated otherwise, please design a deterministic algorithm.

▷ If not stated otherwise, please assume that n to be the size (the number of bits) of the input instance.

▷ If not stated otherwise, please assume that the graphs under consideration are finite, weighted, and directed graphs which does not contain any self loop.

▷ We have not defined some standard problems. You are expected to find the problem statements (from Google say) by yourself.

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1. Let n denotes the number of vertices in the graph. Prove that if there exists an k^n factor approximation algorithm for the Travelling Salesman problem for any positive integer k, then $P = NP$.

2. Design a $\frac{7}{8}$ factor approximation algorithm for the MAX3SAT problem. [Hint: try greedy approach.]

3. Design a $\mathcal{O}(\log n)$ factor approximation algorithm for the Set Cover problem. [Hint: try greedy approach.]

4. Design a 2 factor approximation algorithm for the Vertex Cover problem. [Hint: try to find polynomial time computable lower bound on the size of a minimum vertex cover.]

5. Design a 2 factor approximation algorithm for the Bin Packing problem. [Hint: This is Problem 35-1 in the CLRS book. Follow the guideline provided in CLRS.]

6. Design a FPTAS for the 0/1-Knapsack problem.

7. Prove that if there exists an α factor approximation algorithm for the Maximum Clique problem, then there exists a $\sqrt{\alpha}$ factor approximation algorithm for the Maximum Clique problem for any constant α. Deduce from this that if if there exists an α factor approximation algorithm for the Maximum Clique problem, then there exists a PTAS for the Maximum Clique problem. [Hint: Use hint from Problem 35-2 in the CLRS book.]

8. Prove that if there exists a FPTAS for the Vertex Cover problem, them $\mathsf{P} = \mathsf{NP}$.

9. Design a randomized $\frac{1}{2}$ factor approximation algorithm for the Maximum Cut problem. Deduce from this that for every graph, there exists a cut consisting of at least half the edges in the graph.

10. Design a deterministic $\frac{1}{2}$ factor approximation algorithm for the Maximum Cut problem. [Hint: try greedy approach.]

11. We know that CNF-SAT is NP-complete due to the famous Cook-Levin Theorem. Design a polynomial time algorithm for DNF-SAT.

12. We know that 3SAT is NP-complete. Design a polynomial time algorithm for 2SAT.