INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Autumn Semester 2018-19

Date of Examination:
Session (FN/AN):
Duration: 2 hours
Full Marks: 60
Subject No: CS60007
Subject: ALGORITHM DESIGN AND ANALYSIS
Department/Center/School: COMPUTER SCIENCE AND ENGINEERING
Specific charts, graph paper, log book etc., required: NO
Special instruction (if any): NA

Answer question 5 and any three of the first four questions.

1. [A-perfect r-matching] Let $G = (A \cup B, E)$ be a bipartite graph. An r-matching of G is a subset $M \subseteq E$ of edges such that

   (a) For each vertex u in A, $|\{v \in B : (u,v) \in M\}| \in \{0, r\}$, i.e., each vertex in A is matched to either no vertex in B, or exactly r vertices in B,

   (b) For each vertex v in B, $|\{u \in A : (u,v) \in M\}| \in \{0, 1\}$, i.e., each vertex in B is matched to at most one vertex in A.

   Note that the usual notion of matching corresponds to $r = 1$. An r-matching M is said to be A-perfect if all the vertices in A are matched in M, i.e., for each vertex u in A, $|\{v \in B : (u,v) \in M\}| = r$. Recall that for a subset $A' \subseteq A$, the neighborhood $\mathcal{N}(A')$ of A' is defined to be the set of neighbors of vertices in A', i.e., $\mathcal{N}(A') = \{v \in B :$ there exists $u \in A$ such that $(u,v) \in E\}$. Prove that G has an A-perfect r-matching *if and only if* for each subset $A' \subseteq A$, $|\mathcal{N}(A')| \geqslant r|A'|$. Note that for $r = 1$, this is Hall's theorem.

   [15 Marks]

2. Let G be an undirected weighted graph. Each edge f in G has a real weight $w(f)$ which could possibly be negative. Let T_1 and T_2 be two different minimum spanning trees of G. Let $e = (u,v)$ be an edge that is in T_1 but not in T_2. Let $\mathcal{P}$ be the unique path between u and v in T_2. Show that $\mathcal{P}$ has an edge e' such that $w(e') = w(e)$.

   [15 Marks]

3. Given two strings x and y of lengths m and n respectively over an alphabet Σ, design an algorithm with worst case running time $O(mn)$ to find the edit distance between x and y. The edit distance between any two strings x and y is the minimum number of operations one needs to perform to transform x into y. The following operations are allowed.

   (i) **Insertion:** any symbol from Σ can be inserted at any position in a string.

   (ii) **Deletion:** any symbol from a string can be deleted.

   (iii) **Substitution:** any symbol from a string can be replaced with another symbol.

   [15 Marks]

4. A vertex cover of an undirected graph $G = (V, E)$ is a set of vertices $U \subseteq V$ such that each edge has one of its endpoints in U, i.e., for each edge $(u, v) \in E$, we have $u \in U$ or $v \in U$ (or both). Prove using linear programming duality that if G is bipartite then the size of its maximum matching is equal to the size of its minimum vertex cover.

[15 Marks]

5. [Hamiltonian path] Let G be an undirected graph with n vertices. A Hamiltonian path of G is a path which visits each vertex of G exactly once. Design a $O(\mathsf{poly}(n) \cdot 2^n)$ time algorithm to determine if G has a hamiltonian path, and to find a Hamiltonian path in case G has one, where $\mathsf{poly}(n)$ is any polynomial function of n.

[15 Marks]