CS60050: Machine Learning

Mid-semester Examination, Autumn 2017

Time= 2 hrs. Marks: 45. Answer all THREE questions.

1.(a) Show a decision tree that realizes the parity function of four Boolean variables, *A*, *B, C,* and *D*. A parity function evaluates to 1 if there are odd number of ones in input, and 0 otherwise. [5]

(b) Build a decision tree to classify the following patterns. Use the information gain criterion. *Show all the calculations systematically.* What Boolean function does the tree realize? [10]

Pattern (x1,x2,x3)	Class
(0, 0, 0)	0
(0, 0, 1)	0
(0, 1, 0)	0
(0, 1, 1)	0
(1, 0, 0)	0
(1, 0, 1)	1
(1, 1, 0)	0
(1, 1, 1)	1

2. Consider two classes ω_1 and ω_2. We want to classify a variable *x* into one of these two classes. Suppose $p(x|\omega_1)$ and $p(x|\omega_2)$ are defined as follows: [10 + 5]

$$p(x|\omega_1) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}, \forall x$$

$$p(x|\omega_2) = \frac{1}{4}\ , -2 < x < 2$$

(a) Find the minimum error classification rule $g(x)$ for this two-class problem, assuming $p(\omega_1) = p(\omega_2) = 0.5$.

(b) There is a prior probability of class 1, designated as π_1^*, so that if $p(\omega_1) > \pi_1^*$, the minimum error classification rule is to always decide ω_1 regardless of x. Find π_1^*. There is no π_2^* so that if $p(\omega_2) > \pi_2^*$, we would always decide ω_2. Why not?

3. (a). Consider a support vector machine whose input space is $\mathbb{R}^2$, and in which the kernel function is computed as, $k(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x} \cdot \boldsymbol{y} + 1)^2 - 1$, (bold letters represents vectors in $\mathbb{R}^2$). Find the mapping $\phi(\boldsymbol{x})$ to the feature space corresponding to this kernel. Show your derivation. [5]

(b) Let X_1 = *(1,−1,−1)*, y_1 =*−1*, X_2 = *(−3,1,1)*, y_2 = *1*, X_3=*(−3,1,−1)*, y_3=*−1*, X_4= *(1,2,1)*, y_4 =*−1*, and X_5 = *(−1,−1,2)*. y_5 = *1*, be five binary labeled training examples. These points are linearly separable. Derive the optimum margin classifier (support vectors, weights and threshold value) and the margin. [10]

---------- BEST WISHES ---------