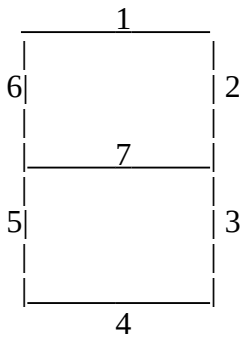
Neural Networks:

1. Draw a perceptron that captures the 3-of-5 concept. In the 3-of-5 concept, there are 5 inputs (x_1, x_2, x_3, x_4, x_5). Each input can either be a 1 or a -1. If 3 or more of the input lines contain a 1, the perceptron should output a 1. Otherwise the perceptron should output a -1.

2. Consider networks whose neurons have *linear activation functions*, i.e., each neuron's output is given by $g(x) = bx+c$, where x is the weighted sum of inputs to the neuron, and b and c are two fixed real numbers.

a) Suppose you have a single neuron with a linear activation function g as above and input $\mathbf{x} = x_0,\ldots,x_n$ and weights $\mathbf{W} = W_0,\ldots,W_n$. Write down the squared error function for this input if the true output is y.
b) Write down the weight update rule for the neuron based on gradient descent on the above error function.
c) Now consider a network of linear neurons with one hidden layer of m units, n input units, and *one output unit*. For a given set of weights w_{kj} in the input-hidden layer and W_j in the hidden-output layer, write down the equation for the output unit as a function of w_{kj}, W_j, and input $\mathbf{x}$. Show that there is a single-layer linear network with no hidden units that computes the same function.

3. Consider the decimal digits 0-9 constructed using a 7 segment display. The seven segments begin at the top (horizontal) and move around the display, with the middle segment being the last.



A digit is represented as a binary string. Thus

0000000	nothing – not legal
1111110	0
0110000	1
1101101	2
1111001	3

etc.

a.) Using only a single-layer perceptron network, can you devise a scheme so that only two output values (a single output node) are given. A 1 if the number (binary string) is even or a 0 if the number is odd.
b.) Using a two-layer backpropagation network, can you devise a scheme so that the single output is the decimal equivalent of the binary input?

4. Construct by hand a neural network for classifying Boolean inputs whose sign agrees with the product of the inputs. In other words, the output y of the neural net must satisfy the following conditions. If $x_1 \cdot x_2 > 0$, then $y>0$., If $x_1 \cdot x_2 < 0$, then $y<0$.

5. Show all steps of the calculations with respect to a single-neuron with a sigmoidal nonlinearity. Assume that you are at the output stage of the network. The objective is for the unit to learn a single input pattern, namely

$$i = \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

The desired output is o = 1. Initially assume $w_1 = w_2 = 0$. Use a learning rate $\eta = 1.0$. *Show all the calculations for two iterations*. Show the weight values at the end of the first and second iterations. In what direction is the weight vector moving from iteration to iteration?