Advanced Machine Learning
Quiz I

1. Let x be a random variable that can take on values $1, 2, \ldots, N$. Suppose you have n i.i.d. observations $x_1, x_2, \ldots, x_n$ that come from the likelihood distribution $f(x|\theta) = \theta(1-\theta)^{\sum_{i=1}^{n} x_i}$. Let the prior distribution of θ be $Beta(a, b)$ with fixed parameters a, b. Show that the posterior distribution is also a $Beta$. What are the parameters of the posterior distribution?

(The $Beta$ distribution has the form $Beta(a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\theta^{a-1}(1-\theta)^{b-1}$.)