

Advanced Machine Learning
Class Test I
Time: 1 hour. Marks: 30

1. Show that the Beta distribution is a conjugate prior for the Bernoulli likelihood distribution. The probability density function for the Beta distribution is $f(x) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)}$; $0 \leq x \leq 1; p, q > 0$, where p and q are the shape parameters, and $B(p, q)$ is the beta function.

[5]

2. Consider a $n \times n$ pixel binary image of handwritten digits. Each pixel values takes on a value 1 (white) or 0 (black). The image consists two regions - a foreground representing the digit, and a background. The foreground pixels are usually black (0), and background is mostly white (1). However, due to noise sometimes foreground regions may contain few white pixels and vice versa.

(a) Model the image pixels as a latent variable model under the following scenarios.

Scenario 1: Each pixel is independent of other pixels, i.e., no spatial correlation,

Scenario 2: Every pixel value depends on values of all other pixels in the image.

For each of the models describe: the random variables involved (including their domain), their distributions, parameters, and the plate diagram.

[5]

(b) We want to estimate the parameters of the models using the EM algorithm. Derive the parameter update rules for the Expectation and Maximization steps.

[10]

(c) The following dataset of two 3×3 pixel images of the handwritten digit '1' is given to you. Make an initial guess about the model parameters. Write down the updated parameter values for one iteration of the EM algorithm for Scenario 1 (independent pixels).

[10]

