

Advanced graph theory: Tutorial 1: CS60047

Autumn August 11, 2023

1. Show that at least two of all persons attending a party must have the same number of friends amongst the attendees.
2. Show that the number of pairs of friends is the half the sum of the numbers of friends of all persons.
3. Show that a k -regular bipartite graph has a perfect matching.
4. Draw the 8-vertex 16-edge complement of the cube of 8 vertices of 12 edges.
5. Draw the Petersen graph P_{10} of $\binom{5}{2}$ vertices, where the vertices correspond to the 2-subsets of the first 5 natural numbers, and the edges correspond to disjoint pairs of such 2-subsets.
6. Find the size $\alpha(P_{10})$ of the largest independent set in the Petersen graph, and the smallest vertex cover of size $\beta(P_{10})$.
7. Show that a graph with girth 5 and minimum vertex degree $\delta \geq k$ has at least $k^2 + 1$ vertices.
8. For $k = 2$ and $k = 3$ find examples of graphs of girth 5 and minimum vertex degree k with exactly $k^2 + 1$ vertices.
9. Show that the Petersen graph is triangle-free.
10. Count the number of 5-cycles in the Petersen graph.
11. Show that all longest paths in a tree pass through a common vertex.
12. Show that this is not generally true for general graphs by giving an example.
13. Show that there are $n - k$ distinct paths of length k in a tree of diameter $2k - 3$.