Assignment 3: CS60023: Approximation and online algorithms

Spring 2024
You must answer all questions totalling 100 marks.
All proofs and arguments must be complete and clear.
Follow lucid writing style, using suitable notation and maintaining rigour.

March 2, 2024

1. For $G(V, E)$ be an undirected vertex weighted graph with weight $w(v)$ at the vertex v and weight $w(X)$ for the sum of weights $w(x)$ for $x \in X \subseteq V$, for the subset X of vertices.

   Let P, Q, R be the sets of vertices of G with solution values $1, \frac{1}{2}$ and 0, for the indicator variables of vertices in the respective sets P, Q, R, in an optimal primal solution of the relaxation of the primal integer LP.

   (i) Show that there is one such rational optimal solution as stated above, for the primal relaxation LP of the primal integer program for this weighted vertex covering problem. [10 marks]

   Now assume that at least one optimal vertex cover for G contains P and each vertex of R has all its neighbours in P. Also, assume that vertex each cover for G has weight lower bounded by $w(P) + \frac{1}{2}w(Q)$. Suppose we do a proper k-coloring of the vertices in G.

   (ii) Show that we can find a vertex cover for G with weighht at most $(2 - \frac{2}{k})OPT$, where OPT is the optimal weight of weighted vertex covers. [10 marks]

2. State the PCP characterization for the class NP. [10 marks]

   Use this characterization to show that $MAX3SAT$ does not have a polynomial time ρ-approximation algorithm for some $\rho < 1$. [15 marks]

   Determine the value of $\rho > 0.5$ such that there is deterministic polynomial time ρ-approximation algorithm for MAX3SAT. [15 marks]

3. Work out the details of the "double coverage" strategy in the K-server problem for online moves to derive the competitive ratio of K using the suitable potential function. Work out the entire derivation. [20 marks]

4. What is the difference between the 3- and 4- factor approximation algorithms for the uncapacitated facility location problem? Analyze how the triangle inequality is used to reduce the number of open facilities. Discuss the way relaxed complementary slackness conditions can help in the design and analysis. Finally, argue the improved 3-factor bound. [20 marks]